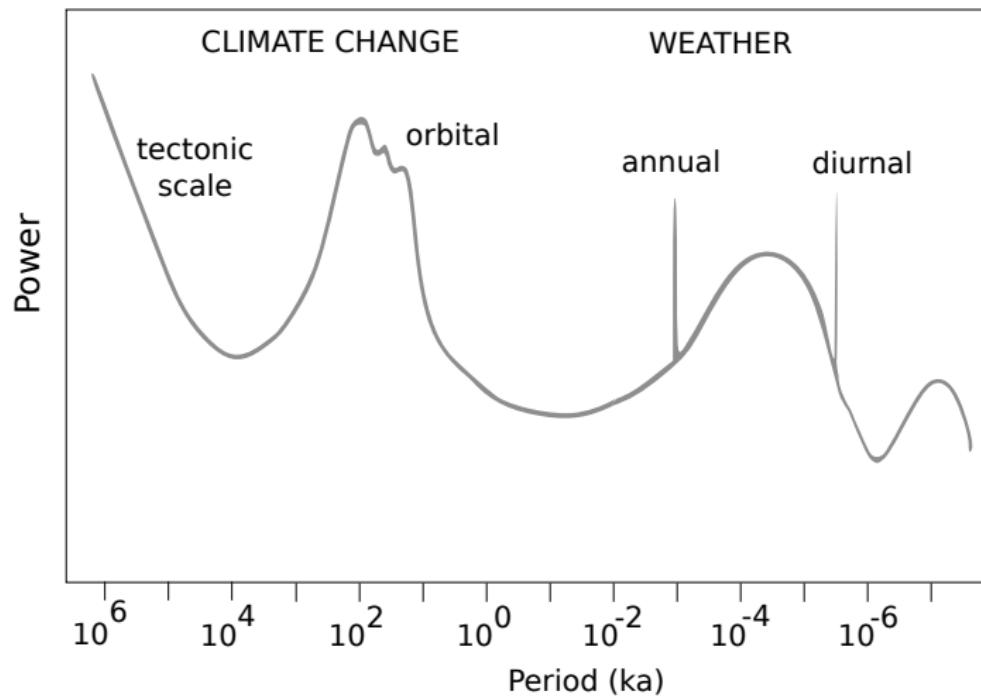


The challenge of centennial variability

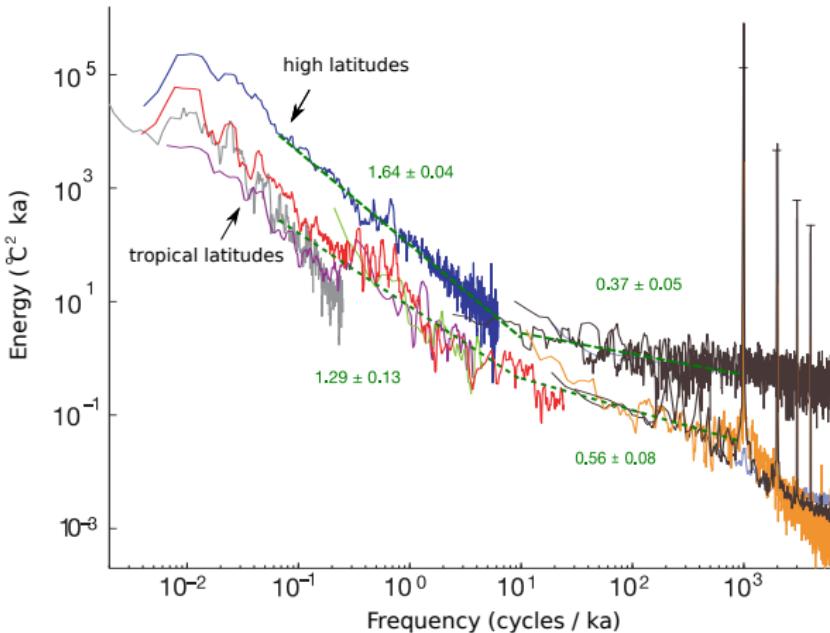
Michel Crucifix

Paleodyn on-line course - December 18th, 2020

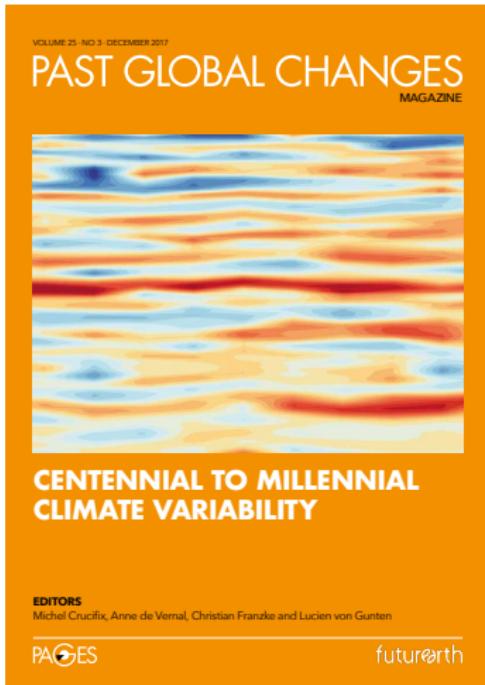
In his “three basic problems of palaeoclimate modelling...” Saltzman suggested a clear time scale separation



Today we rather contemplate a (possible) change in spectral slope



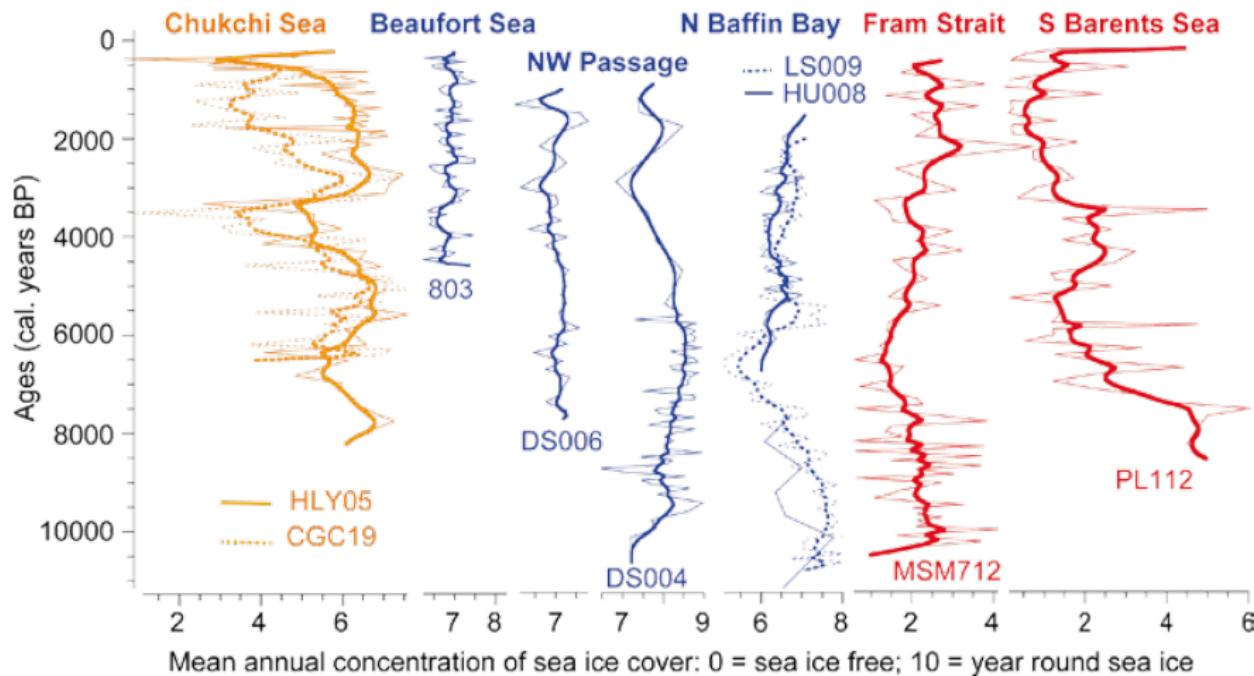
What is going on this plateau ? Why does it occur ?



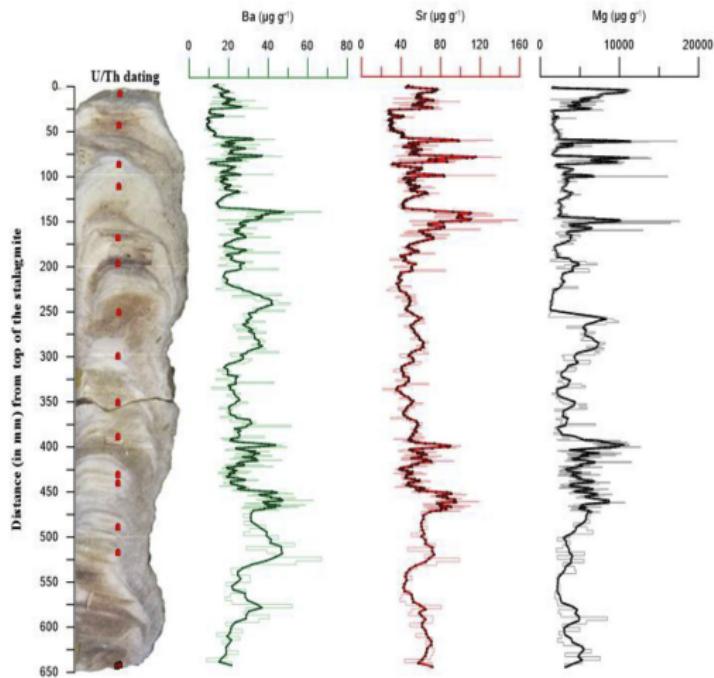
PAGES - 25 (3):
Centennial to Millennial
Climate Variability
(2017), herafeter
PAGES(2017)

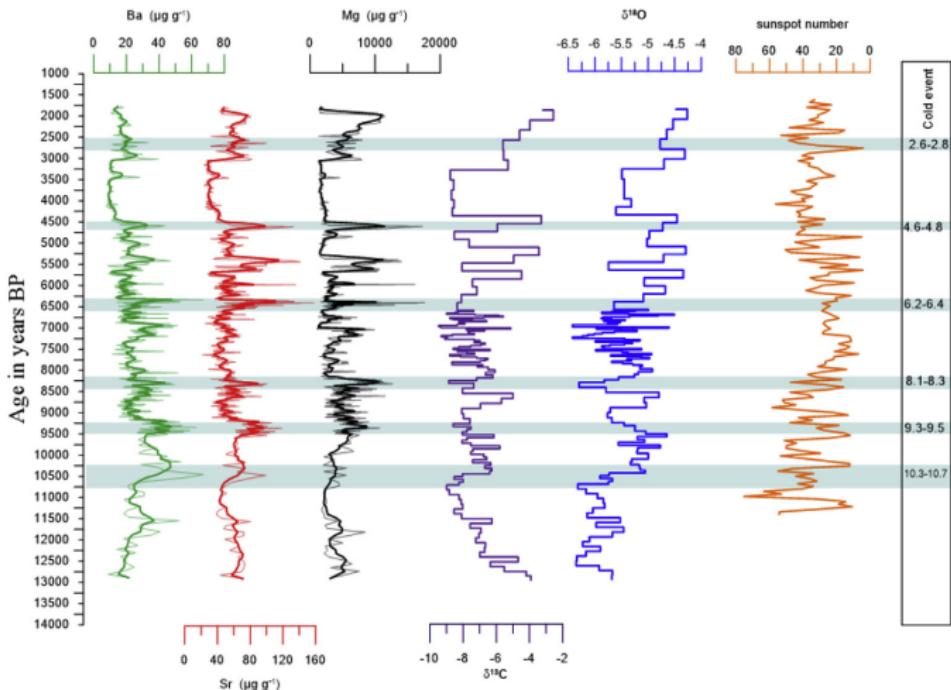
Holocene records can be very detailed, but difficult to decipher

De Vernal. in PAGES (2017)



A speleothem record from Belgium (Pere Noel Cave)





The authors used spectral analysis to detect a possible solar influence

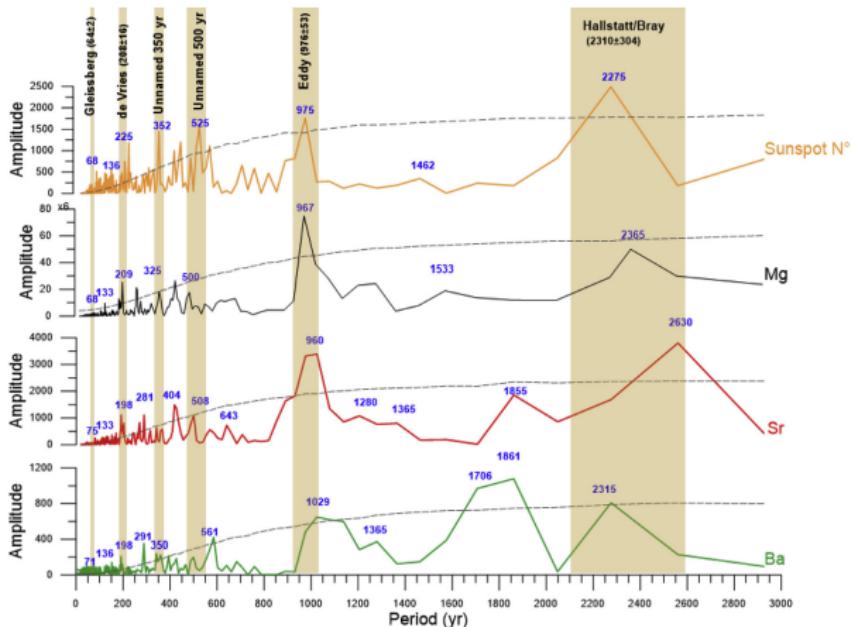
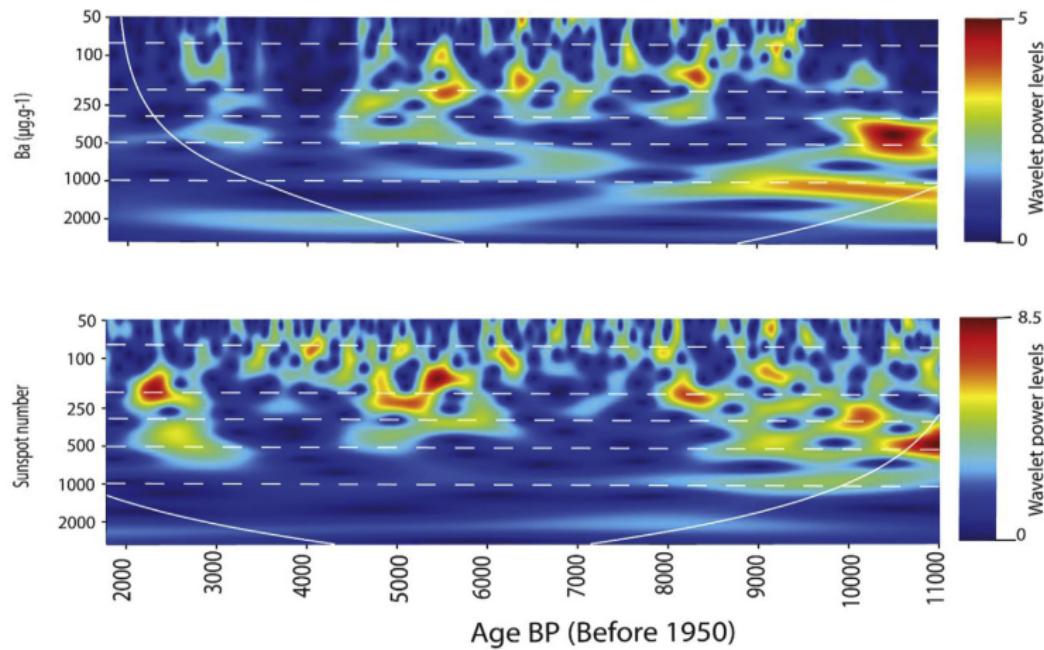
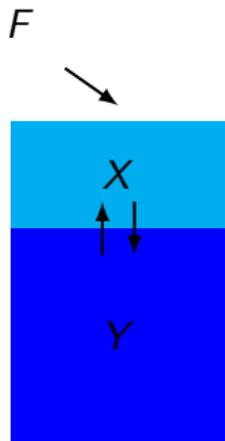


Fig. 5. Intensity of cycles in PN trace elements and sunspot activity (Solanki et al., 2004). The power spectra demonstrate that the strongest cycles of PN trace elements are with durations of 68e 75, 133e 136, 198e 209, 291e 358, 404e 602, 912-1029 and 2365e 2670 yr. The dashed gray lines are the confidence levels (80%), each of those established by 10000 Monte Carlo runs.

Continuous wavelet transform is a common method to detect cycles. But do we use it properly?

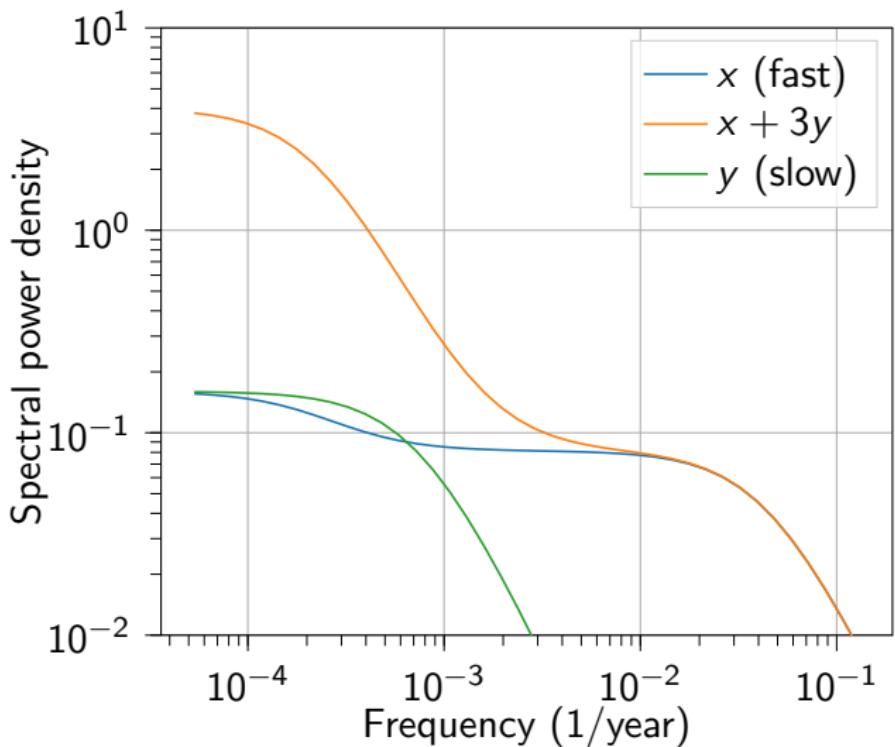


The linear two-box energy balance model

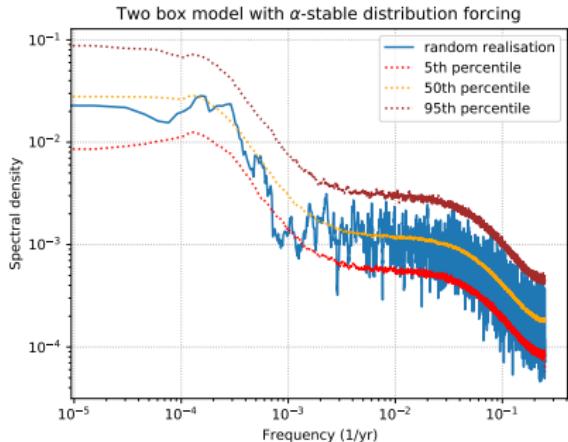
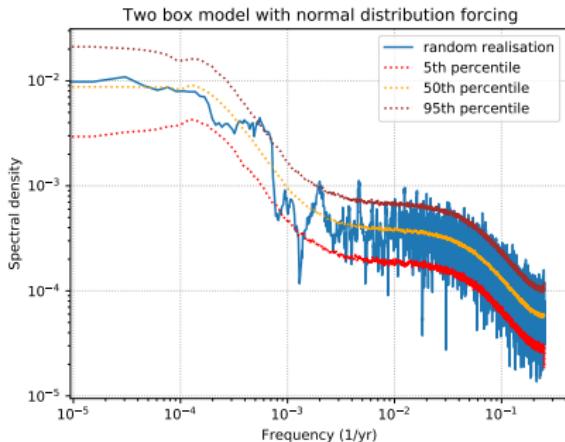


$$\begin{aligned}\frac{1}{\alpha} \frac{dx}{dt} &= -\alpha x + \lambda(y - x) + F(t) \\ \frac{1}{\beta} \frac{dy}{dt} &= -\lambda(y - x)\end{aligned}$$

A plateau emerges from the two distinct time scales

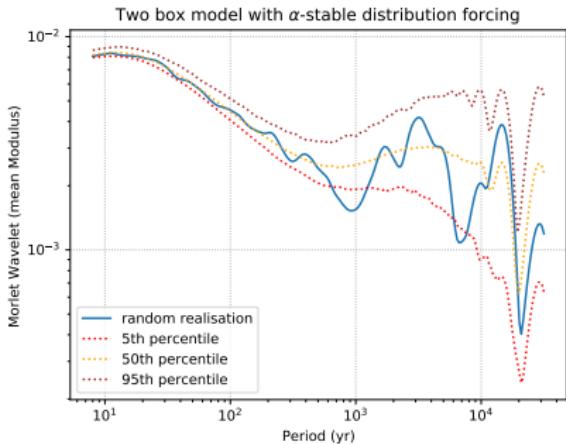
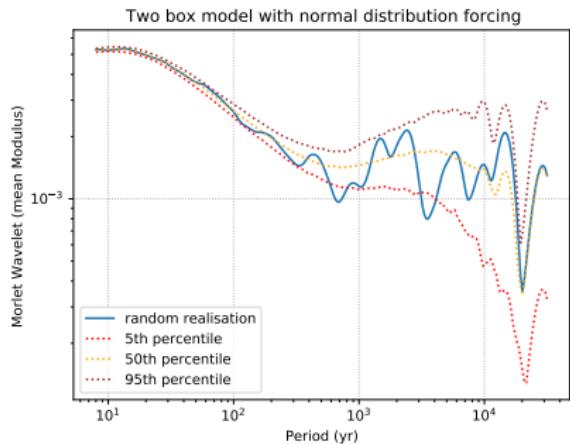


Spectral estimators are expected to reveal spurious peaks.
Watch your null hypothesis



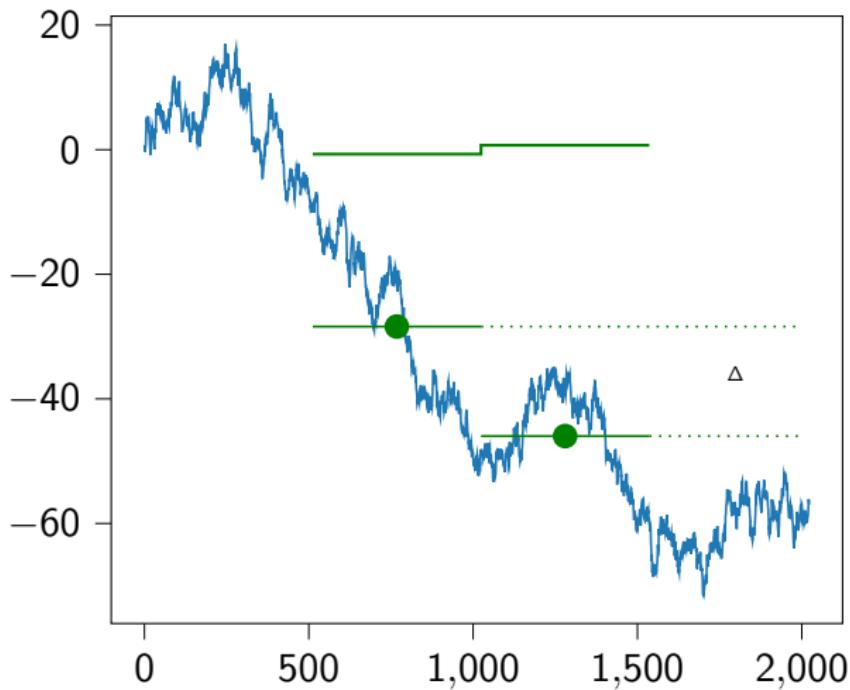
(Walden and Parcival MTM method, using the CRAN package "multitaper")

The Morlet wavelet variance analysis easily generates peaks



(gtseries R package, by M. Crucifix)

The Haar (discrete) wavelet helps to distinguish
'relaxation' regimes from 'wandering regimes'



The Haar (discrete) wavelet helps to distinguish
'averaging' regimes from 'wandering regimes'

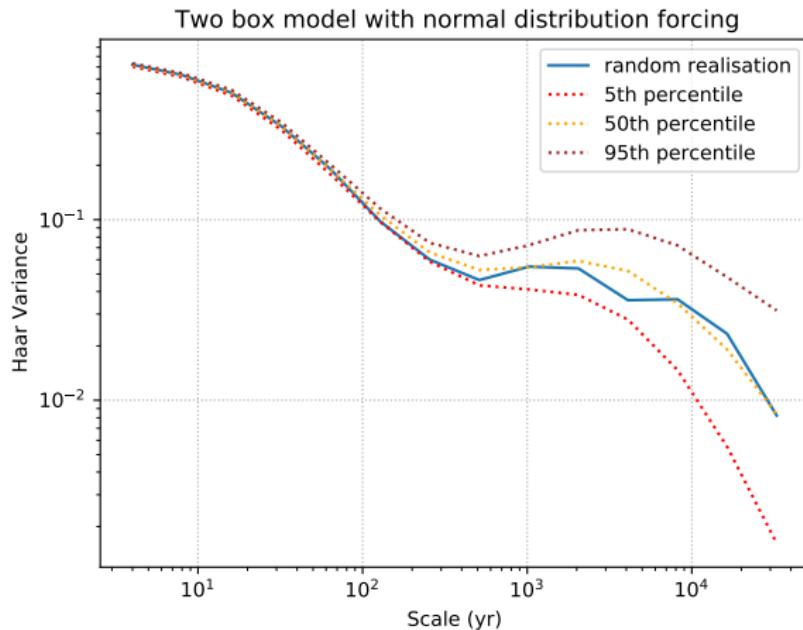
- ▶ Haar variance increasing with scale: "wandering regime" :
- ▶ Haar variance decreasing with scale: "averaging regime" : you can define an average on these time scales

The Haar (discrete) wavelet helps to distinguish 'averaging' regimes from 'wandering regimes'

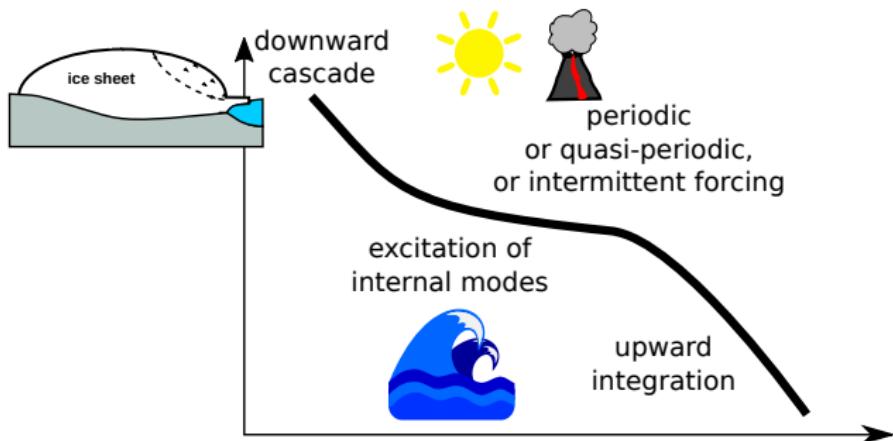
- ▶ Haar variance increasing with scale: “wandering regime” :
- ▶ Haar variance decreasing with scale: “averaging regime” : you can define an average on these time scales

- ▶ **@Lovejoy13ab** proposes to call “macro-” regimes the averaging regimes. “Macroweather” (100-yr scale), “Macro-climate” (1 My time scale).

The Haar wavele on the two-box model

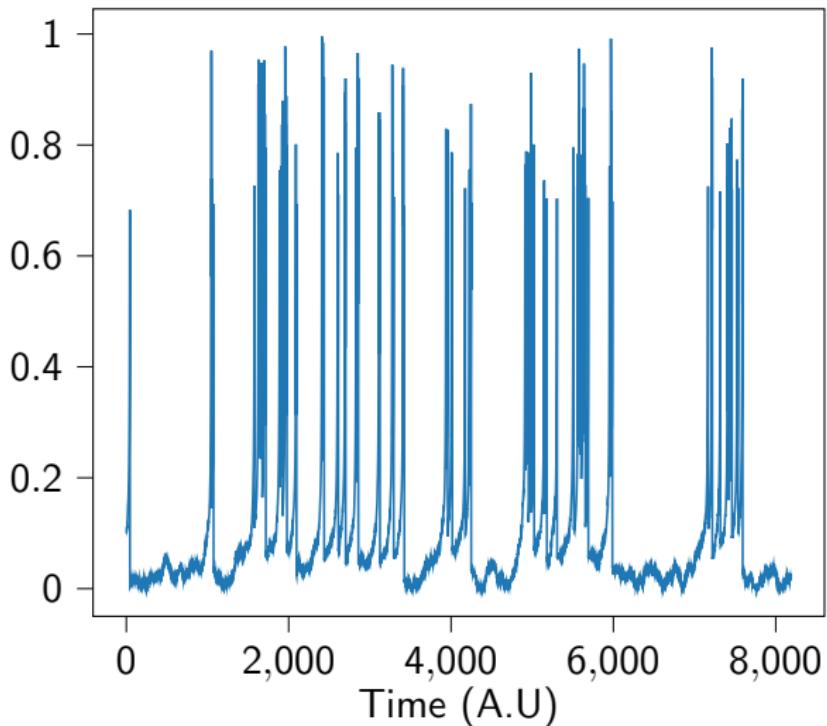


Beyond the linear effects of atmospheric (white) noise:



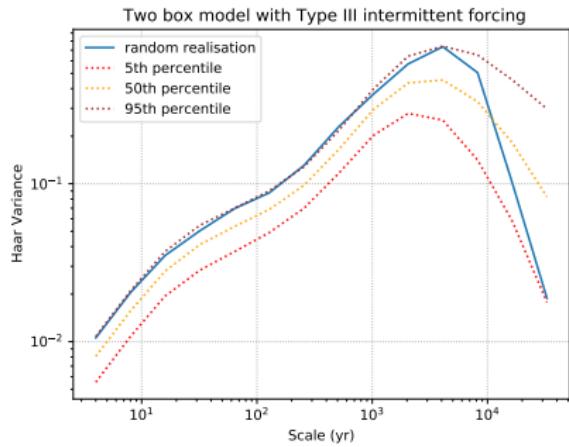
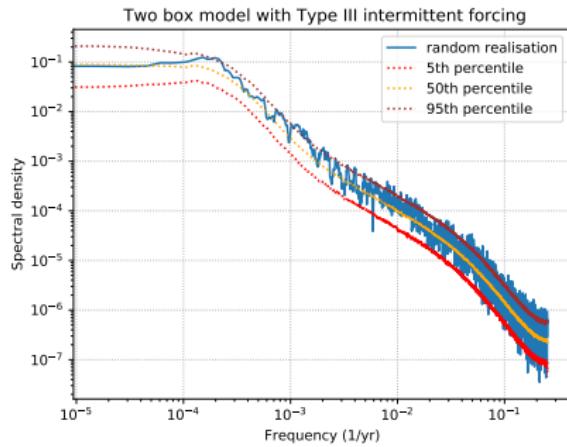
Simulating an intermittent regime: the Pomeau-Manneville Type III intermittency

Type III intermittent

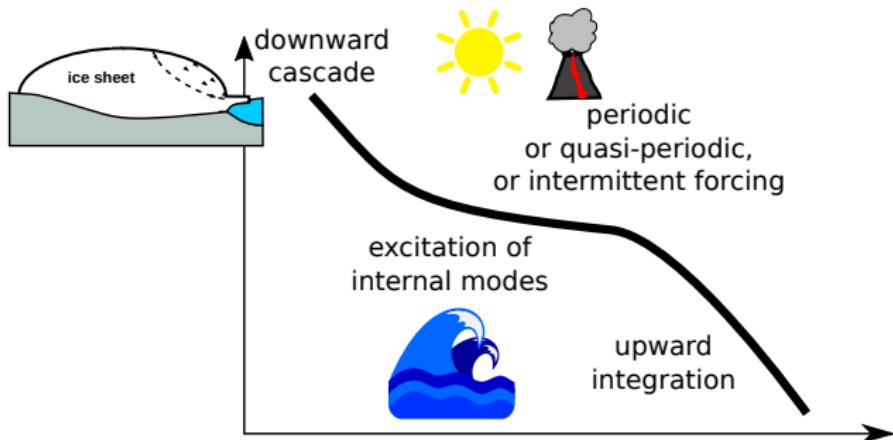


- Simple (5-lines) algorithm -
Distributes variance on a wide range of (Haar) time scales

Exciting the two box model with Type III intermittency : excite the long-term variance



Specific modes at the centennial time scale ?



The oceanic loop oscillation

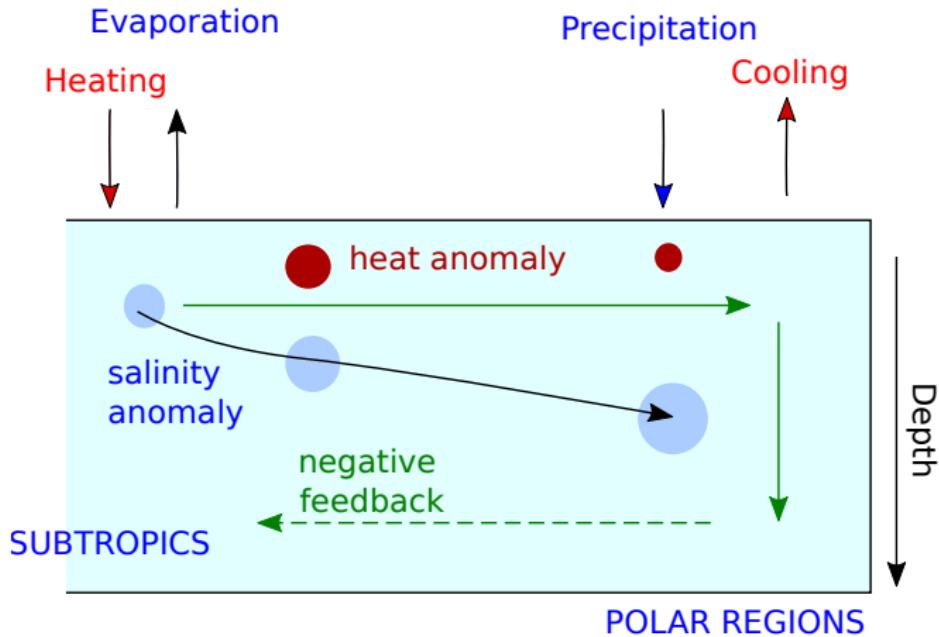
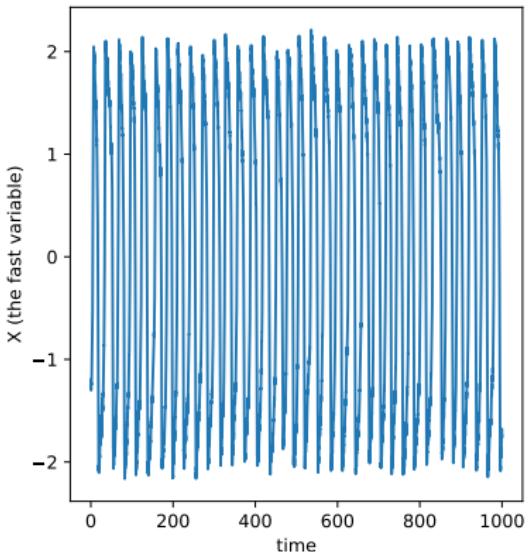
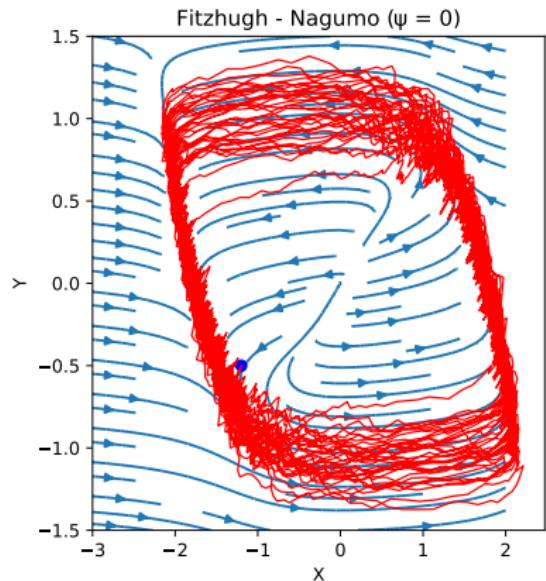


Figure redrawn from Dijkstra and von der Heydt in PAGES (2017), which was reproduced in von der Heydt et al., 2021.

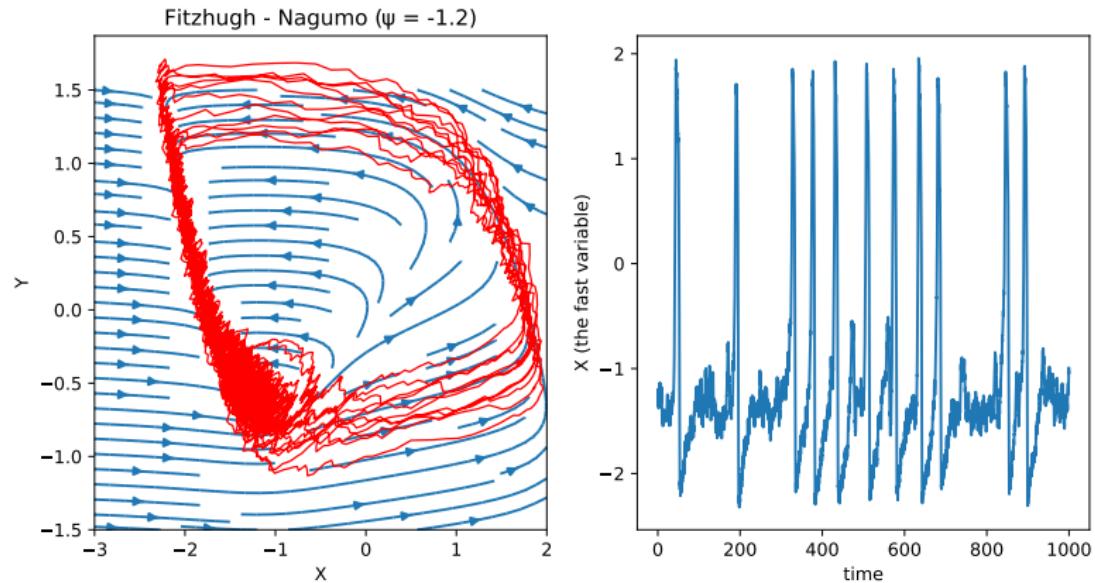
The Fitzhugh-Nagumo ODE is a simple oscillation

$$\begin{aligned}\frac{dx}{dt} &= -x^3/3 + 2x - \lambda(y - x) + F(t) \\ \frac{1}{\gamma} \frac{dy}{dt} &= -\lambda(0y - x - \psi)\end{aligned}$$

It displays a limit cycle (here with stochastic forcing)



However it may also be brought to an ‘exciting’ regime, providing a mechanism for intermittent behaviour



A stratification oscillation in the Southern Ocean ?

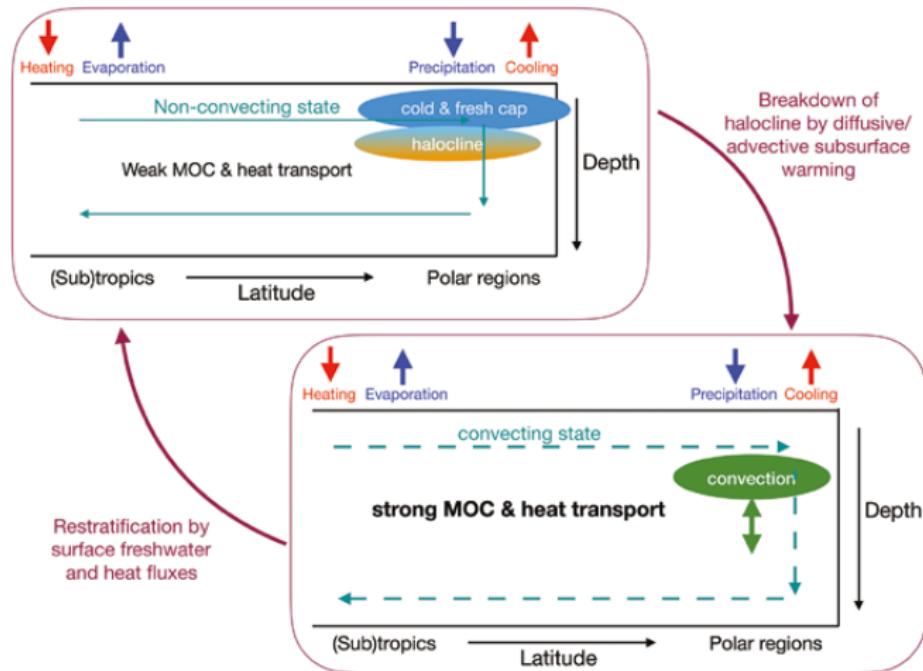
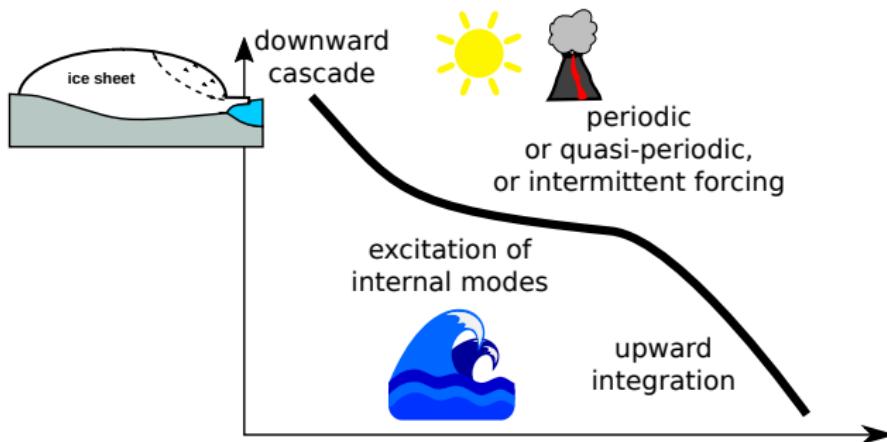


Figure by Dijkstra and von der Heydt in PAGES (2017), based on [M. Winton](#) and [E. S. Sarachik](#). In: *Journal of Physical Oceanography* (1993) and referring to [T. Martin](#), [W. Park](#), and [M. Latif](#). In: *Climate Dynamics* (2012).

Cryosphere's action ?



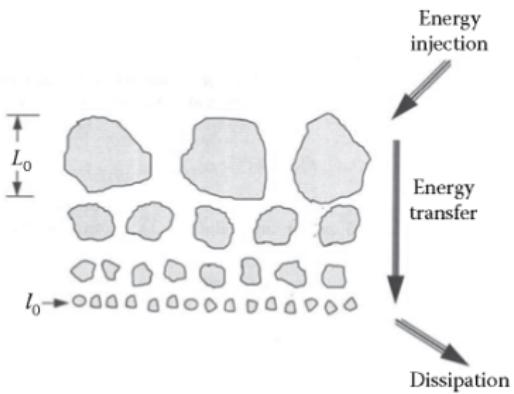
see also Verbitsky and Crucifix (in discussion and accepted in Earth System Dynamics) for a proposal about damped oscillations of small ice sheets

raising complexity

- ▶ So far we have focused on low-dimensional systems.
Things which can be explained with “conceptual models” focusing on exchanges between a small number of boxes

raising complexity

- ▶ So far we have focused on low-dimensional systems. Things which can be explained with “conceptual models” focusing on exchanges between a small number of boxes
- ▶ We know reality is more high-dimensional, and some mechanisms can just no be depicted in this framework



from: https://ebrary.net/58361/engineering/turbulence_energy_cascade_theory

Further reading

- ▶ Our PAGES Newsletter
- ▶ Anna S. von der Heydt et al., Quantification and interpretation of the climate variability record, *Global and Planetary Change*, Volume 197, 2021,