Model selection, parameter and state estimation with uncertain chronology

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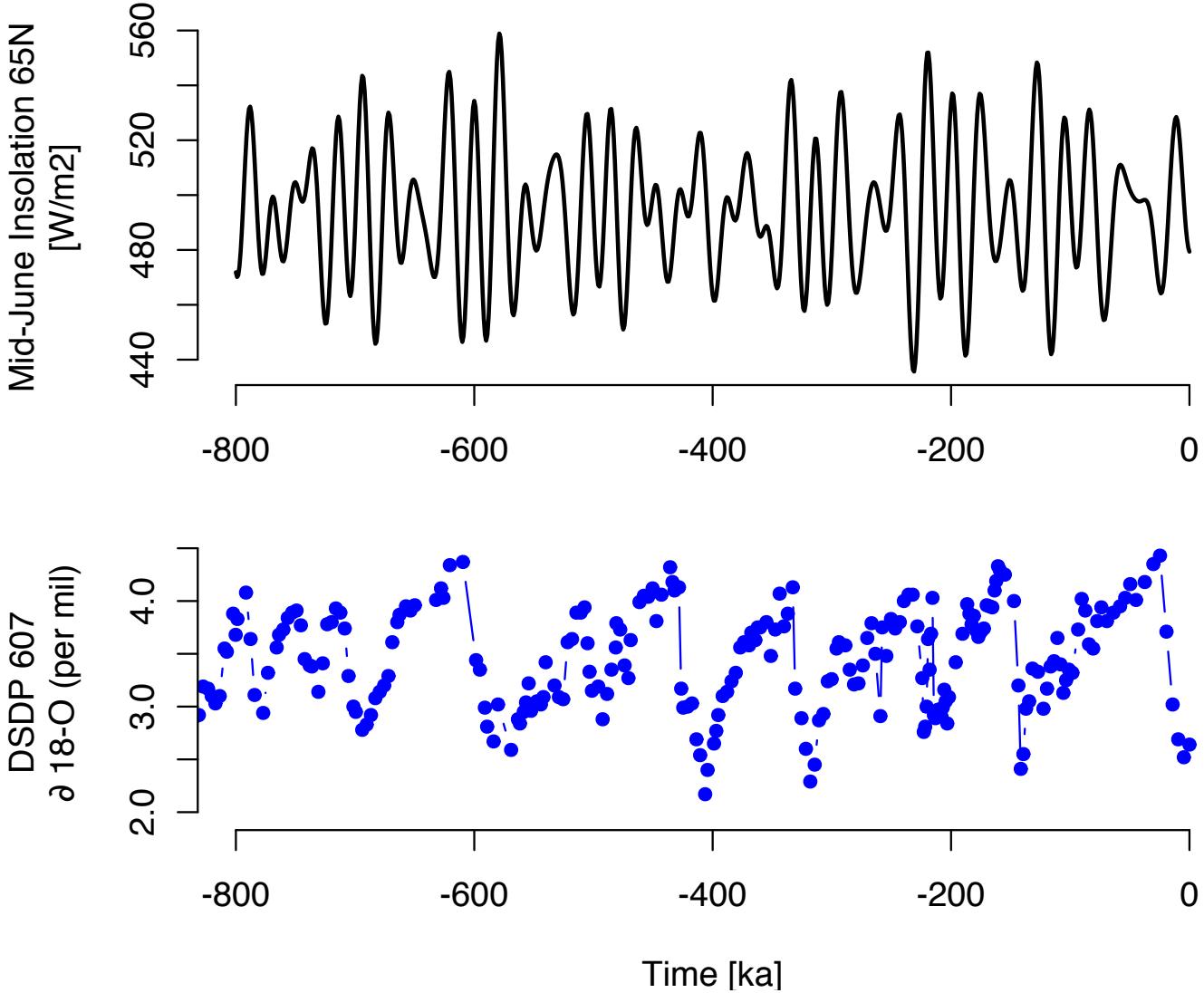
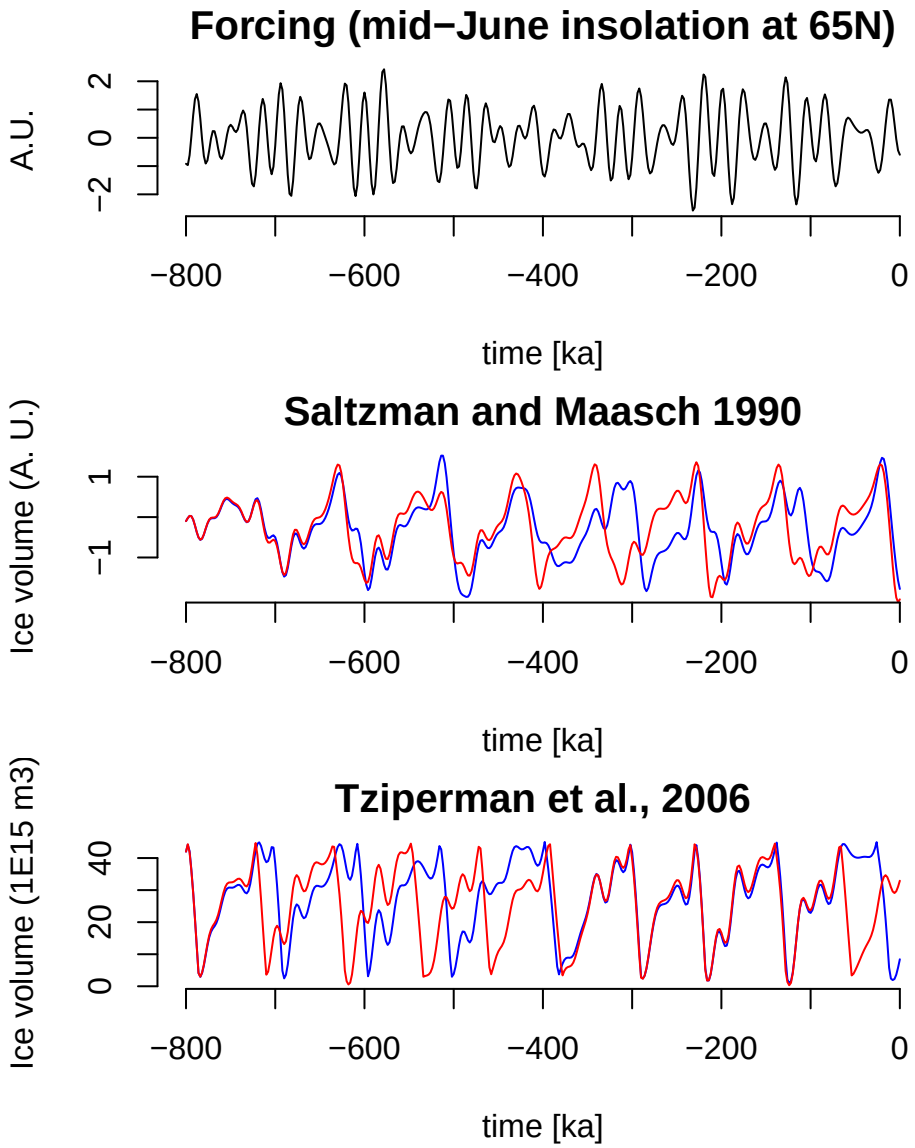
Ice ages



A. L. Berger. In: **Journal of the Atmospheric Sciences** (1978). DOI: 10.1175/1520-0469(1978)035<2362:LTVODI>2.0.CO;2, M. E. Raymo et al. In: **Paleoceanography** (1989).

Ice ages as the output of a (forced, random) dynamical system



Bayesian Model Selection

$$\boxed{\text{Bayes Factor} = \frac{P(Y_{1:M}|\mathcal{M}\text{odel}_1)}{P(Y_{1:M}|\mathcal{M}\text{odel}_2)}} \tag{1}$$

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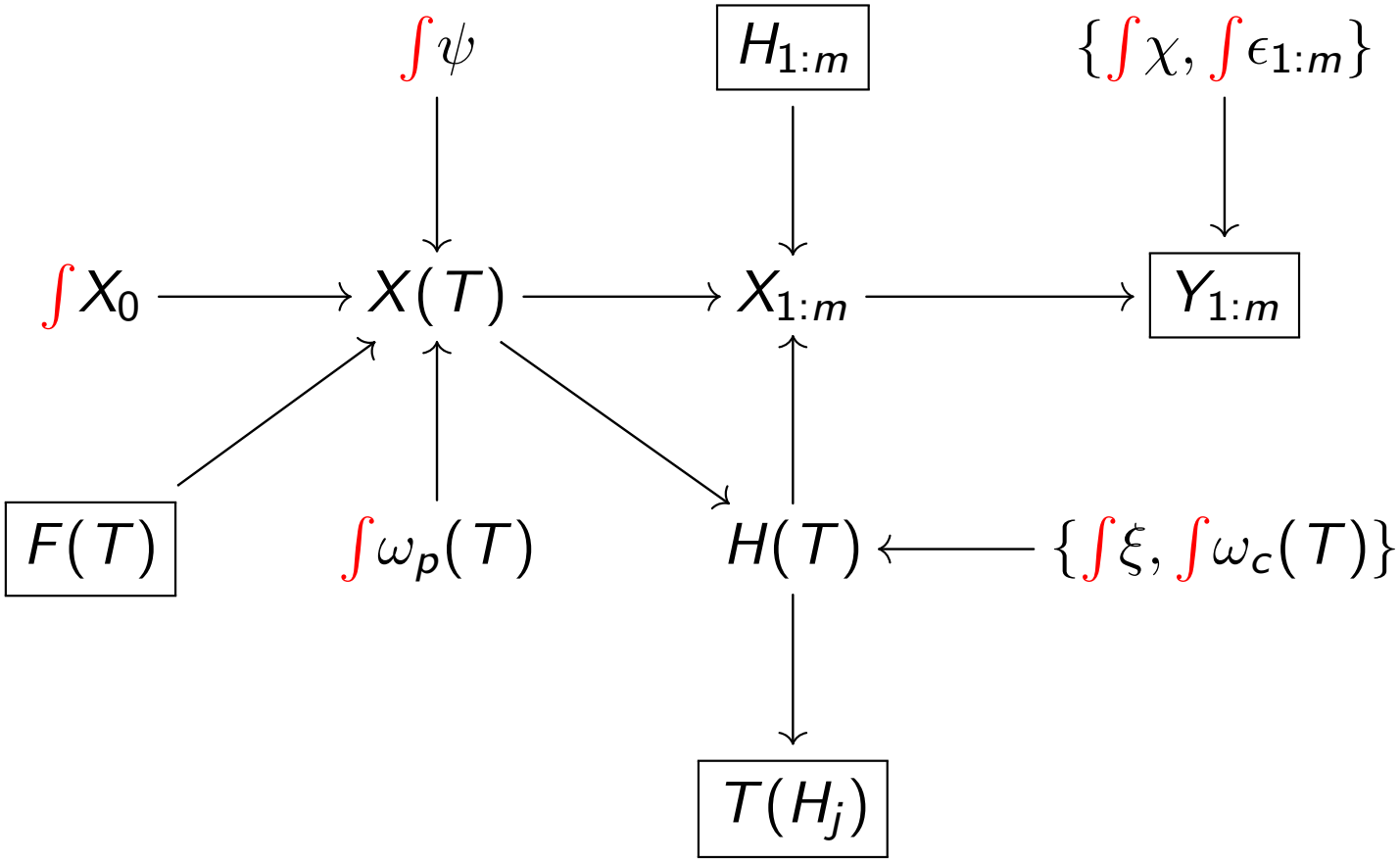
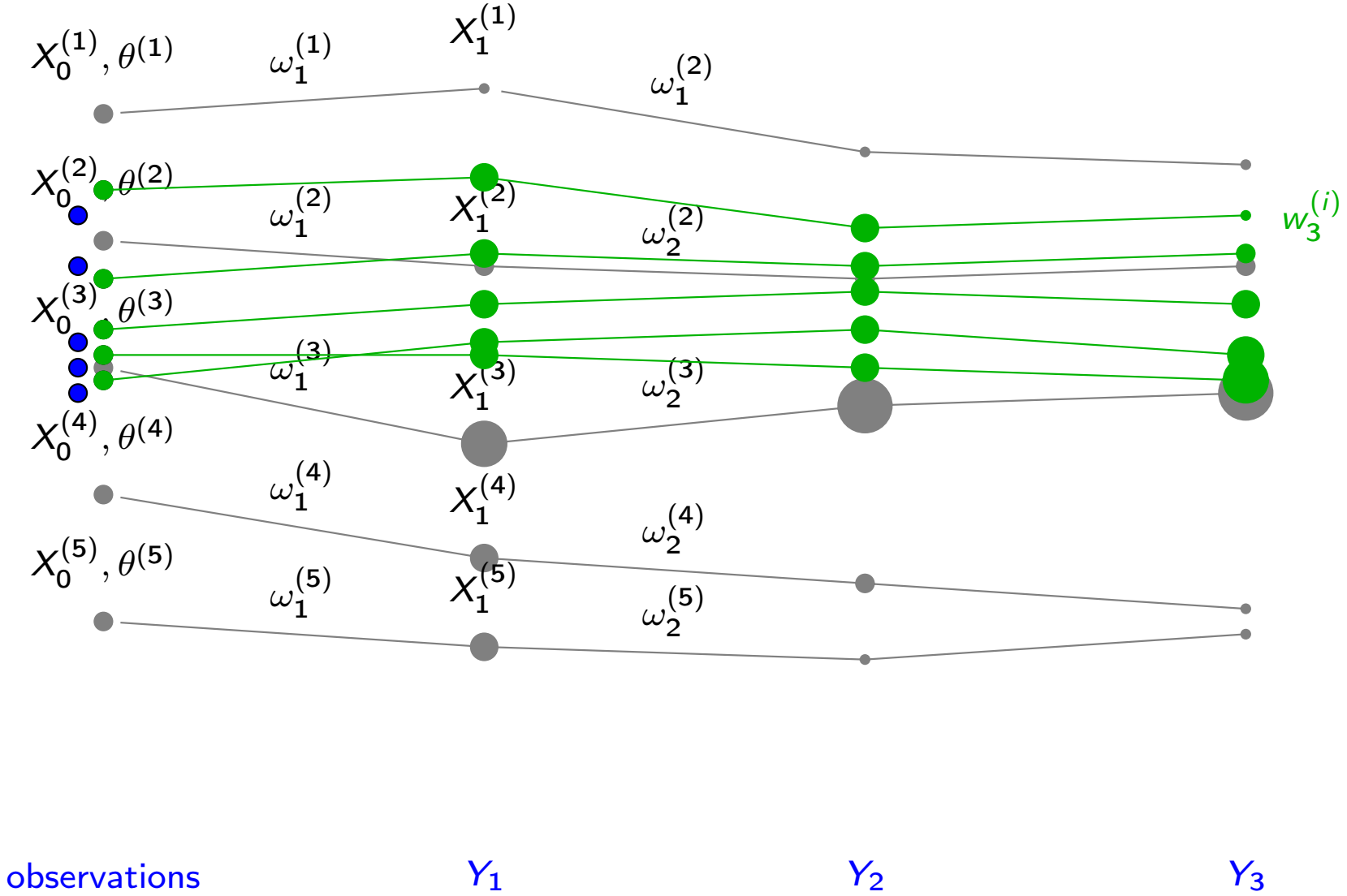
Directed Acyclic Graph (DAG)



Sequential Monte-Carlo Square (SMC2) [Chopin et al. 2012]



$P(Y_{1\ldots3}|\mathcal{M}) \propto \sum w_3^{(i)}$

Testing

- Not shown here for lack of time
- But of course all methods need to be tested with **simulation studies**

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Three ice age models

- CR12

$$
\begin{aligned}
dX_1 &= -\left(\beta_0 + \beta_1 X_1 + \beta_2 \left(X_1 - X_1^3\right) + \delta X_2 + F(\gamma_p, \gamma_e)\right) dt + \sigma_1 dW_1 \\
dX_2 &= \alpha\delta \left(X_1 + X_2 - \frac{X_2^3}{3}\right) dt + \sigma_2 dW_2
\end{aligned}
$$

- SM91

$$
\begin{aligned}
dX_1 &= -\left(X_1 + X_2 + vX_3 + F(\gamma_p, \gamma_e)\right) dt + \sigma_1 dW_1 \\
dX_2 &= \left(rX_2 - pX_3 - sX_2^2 - X_2^3\right) dt + \sigma_2 dW_2 \\
dX_3 &= -q\left(X_1 + X_3\right) dt + \sigma_3 dW_3
\end{aligned}
$$

- T06

$$
\begin{aligned}
dX_1 &= \left(\left(p_0 - KX_1\right)\left(1 - \alpha X_2\right) - \left(s + F(\gamma_p, \gamma_e)\right)\right) dt + \sigma_1 dW_1 \\
X_2 &: \quad 0 \to 1 \text{ when } X_1 \text{ exceeds some threshold } X_u \\
X_2 &: \quad 1 \to 0 \text{ when } X_1 \text{ decreases below } X_l
\end{aligned}
$$

Selection Table: The selected models depends on dating assumpions}

Model		Evidence	
		ODP677: H07(unforced)	ODP677: LR04(forced)
SM91	Forced	4.0×10^{24}	1.1×10^{28}
	Unforced	3.5×10^{26}	1.6×10^{18}
T06	Forced	3.3×10^{25}	4.5×10^{29}
	Unforced	1.7×10^{28}	3.3×10^{21}
PP12	Forced	1.5×10^{22}	1.8×10^{34}

The dating method applied changes the answer

- Using Huybers' non-orbitally tuned data, we find evidence in favour of the unforced T06 model.
- Using Lisiecki's orbitally tuned data, we find strong evidence for PP12 a tuned model (PP12)

Moreover, orbitally tuned data leads us to strongly prefer the orbitally tuned version of each model (and vice versa)

The age model used to date the stack (often taken as a given) has a strong effect on model selection conclusions

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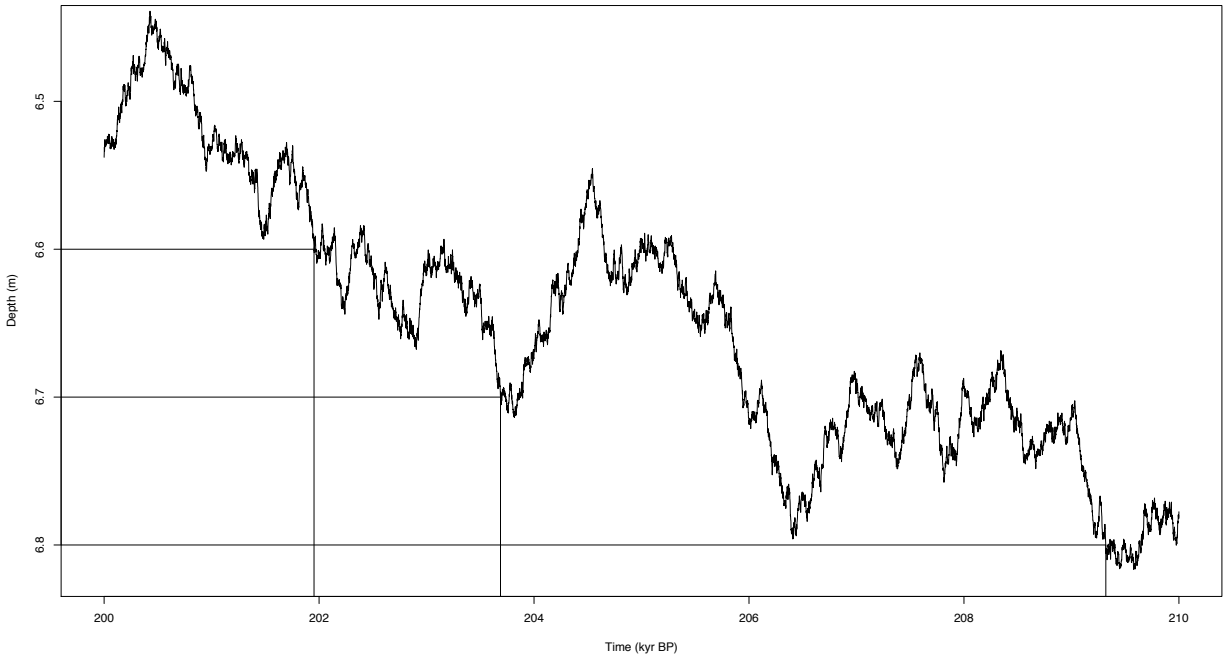
Model

- Mean sedimentation rate + random component

$$\mathrm{d}H = -\mu_s \mathrm{d}T + \sigma \mathrm{d}W \tag{2}$$

- Accounts for sediment compaction similar to Huybers (2007)
- Acconts for dissolution events (age grows monotonously)

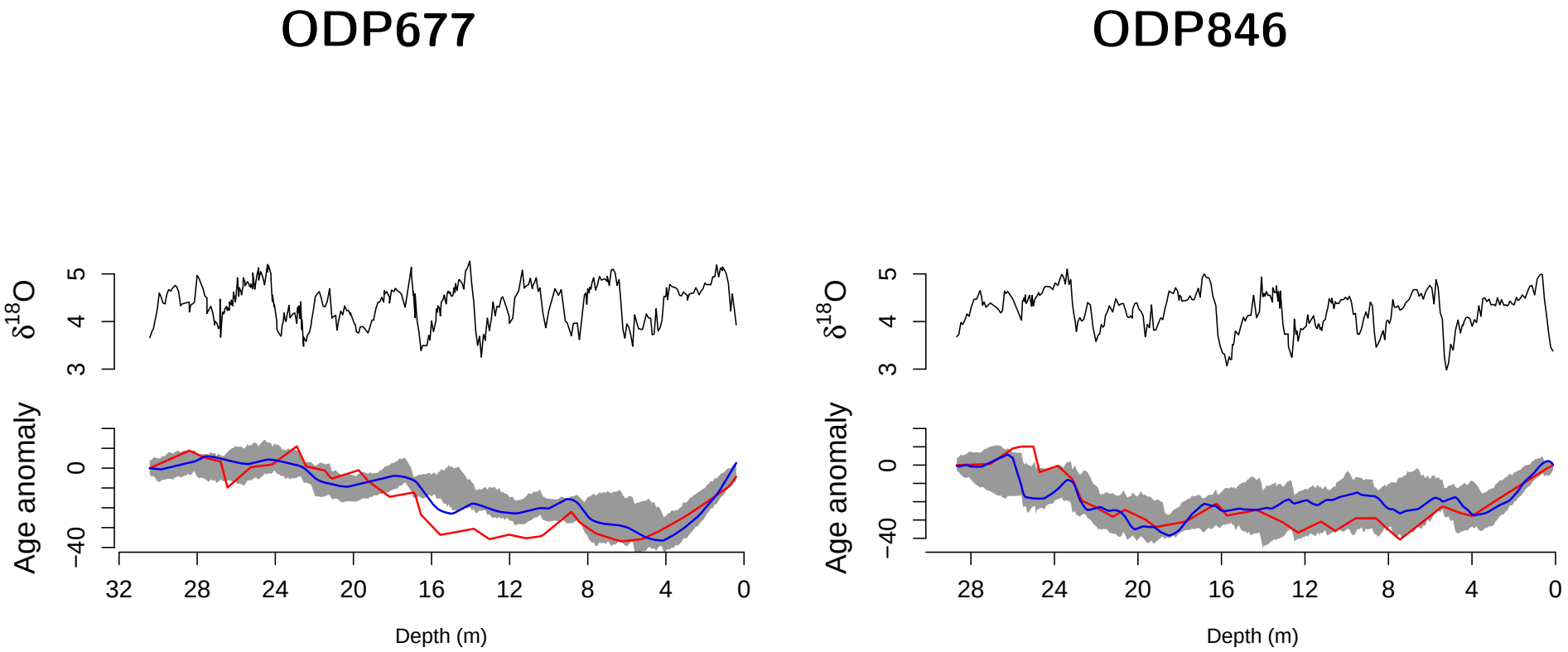
Account for dissolution events



There may have been multiple times when a certain depth was reached: the most recent time is the age of that slice, i.e., it is a first passage problem. Given (T_m, H_m), then T_{m-1} is the first passage time of H_{m-1} with

$$T_{m-1} | T_m \sim IG\left(T_m - \frac{H_{m-1} - H_m}{\mu_s}, \frac{(H_{m-1} - H_m)^2}{\sigma_s^2}\right)$$

Age Estimates



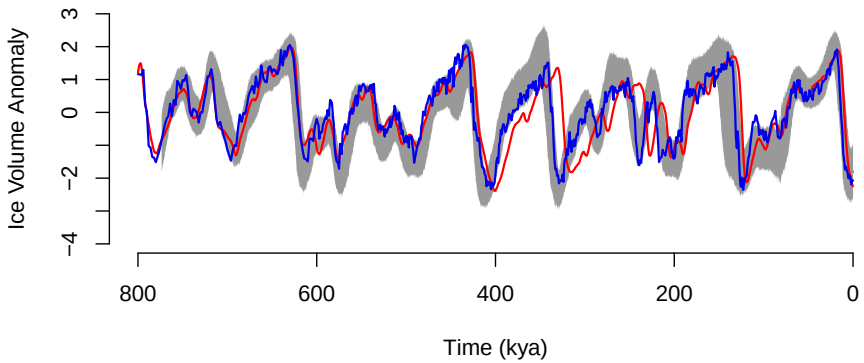
— : as used for Lisiecki & Raymo 2005

— : as used for Huybers 2007

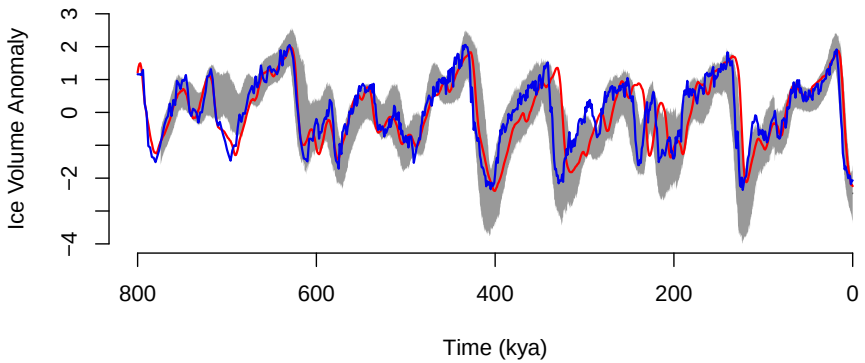
N. J. Shackleton, A. Berger, and W. R. Peltier. In: **Transactions of the Royal Society of Edinburgh-Earth Sciences** (1990), A. Mix, J. Le, and N. Shackleton. In: **Proceedings of the Ocean Drilling Program** (1995). DOI: 10.2973/odp.proc.sr.138.160.1995

State Estimate (consistent for two different cores)

ODP677



ODP846



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Discussion

- Elementary example
  - only one type of data
  - one core
  - about one thousand data points
  - tested for robustness to prior and implementation details
- Yet highly computationaly expensive already (Bayesian framework)
  - 1 week for one model
- Joint age-state-parameter necessary for model selection

References:

- N. Chopin, P. E. Jacob, and O. Papaspiliopoulos. In: **Journal of the Royal Statistical Society: Series B (Statistical Methodology)** (2013). DOI: 10.1111/j.1467-9868.2012.01046.x
- J. Carson et al. In: **Journal of the Royal Statistical Society: Series C (Applied Statistics)** (2017). DOI: 10.1111/rssc.12222

see http://perso.uclouvain.be/michel.crucifix for more on dynamical systems of ice ages.