Model selection, parameter and state estimation with uncertain chronology

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Plan

Introduction

Methodology

Application 1: selection with known chronology

Application 2: uncertain chronology

Discussion
Ice ages

Ice ages as the output of a (forced, random) dynamical system

Forcing (mid-June insolation at 65N)

Saltzman and Maasch 1990

Tziperman et al., 2006
Bayesian Model Selection

Bayes Factor = \frac{P(Y_{1:M} | \text{Model}_1)}{P(Y_{1:M} | \text{Model}_2)} \tag{1}
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Directed Acyclic Graph (DAG)
Sequential Monte-Carlo Square (SMC2) [Chopin et al. 2012]

\[
P(Y_1 \ldots 3 | \mathcal{M}) \propto \sum w_3^{(i)}
\]
Testing

- Not shown here for lack of time
- But of course all methods need to be tested with simulation studies
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Three ice age models

- CR12

\[
\begin{align*}
dX_1 &= - (\beta_0 + \beta_1 X_1 + \beta_2 (X_1 - X_1^3) + \delta X_2 + F(\gamma_p, \gamma_e)) \, dt + \sigma_1 dW_1 \\
dX_2 &= \alpha \delta \left( X_1 + X_2 - \frac{X_2^3}{3} \right) \, dt + \sigma_2 dW_2
\end{align*}
\]

- SM91

\[
\begin{align*}
dX_1 &= - (X_1 + X_2 + vX_3 + F(\gamma_p, \gamma_e)) \, dt + \sigma_1 dW_1 \\
dX_2 &= (rX_2 - pX_3 - sX_2^2 - X_2^3) \, dt + \sigma_2 dW_2 \\
dX_3 &= -q (X_1 + X_3) \, dt + \sigma_3 dW_3
\end{align*}
\]

- T06

\[
\begin{align*}
dX_1 &= (\rho_0 - KX_1) (1 - \alpha X_2) - (s + F(\gamma_p, \gamma_e)) \, dt + \sigma_1 dW_1 \\
X_2 : \quad &0 \rightarrow 1 \text{ when } X_1 \text{ exceeds some threshold } X_u \\
X_2 : \quad &1 \rightarrow 0 \text{ when } X_1 \text{ decreases below } X_l
\end{align*}
\]
## Selection Table: The selected models depends on dating assumpions

<table>
<thead>
<tr>
<th>Model</th>
<th>Evidence</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ODP677: H07(unforced)</td>
</tr>
<tr>
<td>SM91</td>
<td>Forced</td>
</tr>
<tr>
<td></td>
<td>Unforced</td>
</tr>
<tr>
<td>T06</td>
<td>Forced</td>
</tr>
<tr>
<td></td>
<td>Unforced</td>
</tr>
<tr>
<td>PP12</td>
<td>Forced</td>
</tr>
</tbody>
</table>

The dating method applied changes the answer

- Using Huybers’ non-orbitally tuned data, we find evidence in favour of the unforced T06 model.
- Using Lisiecki’s orbitally tuned data, we find strong evidence for PP12 a tuned model (PP12)

Moreover, orbitally tuned data leads us to strongly prefer the orbitally tuned version of each model (and vice versa)

The age model used to date the stack (often taken as a given) has a strong effect on model selection conclusions
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- Mean sedimentation rate + random component

\[
dH = -\mu_s dT + \sigma dW
\]

- Accounts for sediment compaction similar to Huybers (2007)
- Accounts for dissolution events (age grows monotonously)
Account for dissolution events

There may have been multiple times when a certain depth was reached: the most recent time is the age of that slice, i.e., it is a first passage problem. Given \((T_m, H_m)\), then \(T_{m-1}\) is the first passage time of \(H_{m-1}\) with

\[
T_{m-1} | T_m \sim IG \left( T_m - \frac{H_{m-1} - H_m}{\mu_s}, \frac{(H_{m-1} - H_m)^2}{\sigma_s^2} \right)
\]
Age Estimates

ODP677

ODP846

--- : as used for Lisiecki & Raymo 2005

— : as used for Huybers 2007

State Estimate (consistent for two different cores)

ODP677

ODP846
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Discussion

- Elementary example
  - only one type of data
  - one core
  - about one thousand data points
  - tested for robustness to prior and implementation details

- Yet highly computationaly expensive already (Bayesian framework)
  - 1 week for one model

- Joint age-state-parameter necessary for model selection
References:


see [http://perso.uclouvain.be/michel.crucifix](http://perso.uclouvain.be/michel.crucifix) for more on dynamical systems of ice ages.