Dynamical reconstruction of the global ocean state during the Last Glacial Maximum

Fit to, e.g., $\delta^{13}C$ data

Reconstructed AMOC

State estimation


LGM Atlantic Ocean: more stratified, stronger but shallower overturning
Towards paleo-ocean state estimation

• “State”: prognostic variables computed by a numerical model, for example:
  – velocity components
  – temperature or salinity
  – water or carbon isotopes

• “State estimation”: constrained least-squares problem
  – minimize cost function subject to the model-time stepping equations
Lagrange multiplier ("adjoint") method

Forward run
Model results ($V$) depend on control variables ($u$)

\[ V = F(u) \]

Comparison with data
quantified by objective function ($J$)

\[ J = J(V) = J[F(u)] = \sum_i (\text{model}_i - \text{data}_i)^2 \]

Adjusting control variables
Adjoint method tells us the direction!

\[ J = J(u) \]


Best state estimation!

\[ \frac{\partial J}{\partial u} \]
Data

• Modern ocean
  – World Ocean Database (1951-1980)
  – \(\delta^{18}O_{\text{seawater}}\) from NASA GISS database (Schmidt et al., 1999) below 150 m
  – WOCE/GLODAP \(\delta^{13}C\) compilation by Schmittner et al. (2013) below 1000 m
  – First guess of atmospheric forcing from CORE project (Griffies et al., 2009)

• LGM ocean
  – MARGO (2009) SST reconstruction
  – \(\delta^{18}O_{\text{calcite}}\) for Atlantic Ocean (Marchal and Curry, 2008)
  – Compilation of benthic foraminiferal \(\delta^{13}C_{\text{calcite}}\) data by Hesse et al. (2011)
  – First guess of atmospheric forcing provided by Merkel et al. (2010)
Model

- MITgcm (http://mitgcm.org)
  - with simplified oxygen (Dail, 2012) and carbon isotope (Marchal and Curry 2008) parameterizations
  - allows for “Automatic Differentiation (AD)”

(“cubed-sphere” configuration for ocean state estimation)
Objective function

\[ J = J_{\text{data}} + J_{\text{SSH}} + J_{\text{ctrl}}, \]

where:

<table>
<thead>
<tr>
<th>Symbol</th>
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<tr>
<td>( J )</td>
<td>objective or cost function</td>
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<tr>
<td>( J_{\text{data}} )</td>
<td>data-model misfit</td>
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<tr>
<td>( J_{\text{SSH}} )</td>
<td>penalty for drift of global-mean sea-surface height (SSH)</td>
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<td>penalties for deviations of control variables from first-guess values (regularization)</td>
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Carry-over technique (Dail, 2012)

- The longer the forward simulation, the less accurate is the adjoint method, because it is only tangent-linear.
- Need a first guess as good as possible, especially for long state estimations, to prevent a drift of the model state.

Evaluation of objection function over last 10 years

- 20-year preparatory state estimation
- 40-year state estimation
- 80-year state estimation
- 200-year state estimation
- 400-year state estimation
Results

Evolution of $\delta^{13}C_{\text{DIC}}$ cost function contribution (example)

- simulations that lead to decrease of cost
  (all simulations shown)
Evolution of $\delta^{13}C_{DIC}$ cost function contribution (example)

- simulations that lead to decrease of cost
  (unsuccessful simulations left out)

→ carried over to 400-a state estimate
Concentration of northern source water obtained from dye tracer experiments along 35° W: (a) modern state estimate (b) LGM state estimate (Kurahashi-Nakamura et al., 2017, Fig. 6).
Differences between reconstructions and observations:
(a) $\delta^{18}O_c$ for LGM state estimate
(b) $\delta^{13}C_c$ for LGM state estimate
(c) $\delta^{18}O_c$ for LGM after 3000 model years
(d) $\delta^{13}C_c$ for LGM after 3000 model years

Differences $< 0.2\%$ (i.e., the assumed uncertainty) are depicted as gray
Conclusions

1. The MITgcm can be successfully fit to modern as well as the LGM data.
   - The different data sets were compatible with the model and each other within their uncertainties.

2. Compared to the modern state estimate, the reconstructed LGM state was more stratified, with a stronger but shallower overturning.
   - The state estimation also provided a continuous global mapping of the sea surface temperature based on model physics.
Additional slides (not shown)
Hypotheses:

- The glacial NADW was shallower than today.  
  - Cf. Duplessy et al. (1988), Labeyrie et al. (1992), Sarnthein et al. (1994) – but compare to Gebbie (2014)
- The ventilation of the deep Atlantic Ocean was weaker than today.

Benthic \( \delta^{13}C \) (Sarnthein et al., 1994)
Geochemical tracer locations:
(a) modern state estimate
(b) LGM state estimate
Blue dots: $\delta^{18}$O data, red dots: $\delta^{13}$C data (Kurahashi-Nakamura et al., 2017, Fig. 1).
Tracer component:

- $\delta^{18}O_{\text{seawater}}$ treated as passive tracer with fixed boundary conditions at 150 m depth (Dail, 2012)

- Parameterization of $\delta^{13}C_{\text{DIC}}$ for abyssal ocean (depths $> 1000$ m, cf. Marchal and Curry 2008) with fixed $\delta^{13}C_{\text{org}}$ and remineralization time scale
Objective function

\[ J = J_{\text{data}} + J_{\text{SSH}} + J_{\text{ctrl}}, \]

where:

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\[ J_{\text{data}} = \sum_{X} (X_{\text{model}} - X_{\text{obs}})^T W_X (X_{\text{model}} - X_{\text{obs}}), \]

where:

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<tr>
<td>( X )</td>
<td>category of data</td>
</tr>
<tr>
<td>( X_{\text{model}} )</td>
<td>model results</td>
</tr>
<tr>
<td>( X_{\text{obs}} )</td>
<td>observations</td>
</tr>
<tr>
<td>( W_X )</td>
<td>weight matrix (error covariances)</td>
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\[ J_{\text{SSH}} = W_{\text{ssh}}(\text{SSH}_1 - \text{SSH}_0)^2, \]

where:

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<tr>
<td>( W_{\text{SSH}} )</td>
<td>weight factor</td>
</tr>
<tr>
<td>( \text{SSH}_1 )</td>
<td>final value of global-mean sea-surface height</td>
</tr>
<tr>
<td>( \text{SSH}_0 )</td>
<td>initial value of global-mean sea-surface height</td>
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\[ J_{\text{ctrl}} = \frac{N_{\text{data}}}{N_{\text{ctrl}}} \times \left[ (T_0^{\text{adj}} - T_0^{1\text{st}})^T W_{T_0} (T_0^{\text{adj}} - T_0^{1\text{st}}) + (S_0^{\text{adj}} - S_0^{1\text{st}})^T W_{S_0} (S_0^{\text{adj}} - S_0^{1\text{st}}) + \sum_i (F(i)^{\text{adj}} - F(i)^{1\text{st}})^T W_{F(i)} (F(i)^{\text{adj}} - F(i)^{1\text{st}}) + (K^{\text{adj}} - K^{1\text{st}})^T W_K (K^{\text{adj}} - K^{1\text{st}}) + (O_0^{\text{adj}} - O_0^{1\text{st}})^T W_{O_0} (O_0^{\text{adj}} - O_0^{1\text{st}}) + (C_0^{\text{adj}} - C_0^{1\text{st}})^T W_{C_0} (C_0^{\text{adj}} - C_0^{1\text{st}}) + W_\alpha (\alpha^{\text{adj}} - \alpha^{1\text{st}})^2 \right], \]
where:

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<tr>
<td>$N_{\text{data}}$</td>
<td>number of model-data pairs</td>
</tr>
<tr>
<td>$N_{\text{ctrl}}$</td>
<td>Number of control variables</td>
</tr>
<tr>
<td>$T_0, S_0$</td>
<td>initial temperature ans salinity fields</td>
</tr>
<tr>
<td>$F$</td>
<td>atmospheric forcing fields</td>
</tr>
<tr>
<td>$K$</td>
<td>vertical diffusivity</td>
</tr>
<tr>
<td>$O_0, C_0$</td>
<td>initial $\delta^{18}O_w$ and $\delta^{13}C_{\text{DIC}}$</td>
</tr>
<tr>
<td>$W$</td>
<td>weighting matrices</td>
</tr>
<tr>
<td>$W_\alpha$</td>
<td>weighting matrix</td>
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Superscript “adj” means adjusted values (i.e., values in the current iteration), superscript “1st” meant first-guess values (i.e., values in the first iteration).
Estimated SST field:
(a) comparison with the MARGO data (dots)
(b) difference from modern state estimate
(Kurahashi-Nakamura et al., 2017, Fig. 2).
Estimated distribution of $\delta^{18}O$ along 32.5° W:
(a) $\delta^{18}O_w$ in modern state estimate,
(b) $\delta^{18}O_w$ and
(c) $\delta^{18}O_c$ in LGM state estimate.
Dots: observations from Atlantic west of 30° W (Kurahashi-Nakamura et al., 2017, Fig. 3).
Estimated distribution of $\delta^{13}C$ along 20° W:
(a) $\delta^{13}C_{\text{DIC}}$ in modern state estimate,
(b) $\delta^{13}C_{c}$ and
(c) $\delta^{13}C_{\text{DIC}}$ in LGM state estimate.
Dots: observations from Atlantic between 10° W and 30° W for modern and east of 30° W for LGM state estimate (Kurahashi-Nakamura et al., 2017, Fig. 4).
Stream function of the Atlantic Meridional Overturning Circulation (AMOC):
(a) modern state estimate
(b) LGM state estimate
(Kurahashi-Nakamura et al., 2017, Fig. 5).

Max: 16.1 Sv
Max: 21.3 Sv
Atlantic zonal-mean differences (LGM state estimate minus modern state estimate):
(a) salinity
(b) potential density with global mean values subtracted (Kurahashi-Nakamura et al., 2017, Fig. 8).
Reconstructed distributions at 3000 m depth:

(a) $\delta^{18}O_c$ for LGM state estimate
(b) $\delta^{13}C_c$ for LGM state estimate
(c) $\delta^{18}O_c$ for LGM after 3000 model years
(d) $\delta^{13}C_c$ for LGM after 3000 model years

Dots: observations including data in the Indian and Pacific Oceans
Locations of monthly SST data for the modern state estimate:
(a) original data sets
(b) reduced data sets
Color code: number of months that have data
(Kurahashi-Nakamura et al., 2017, Fig. A1).